~ AAS Week 4 Extension Questions ~

① Omitted Variable Bias:

You estimate the following regression model: $Y_i = \beta_0 + \beta_1 X_i + u_i$, to generate an estimate of $\hat{\beta}_0, \hat{\beta}_1$. However, the true data generating process can be given by $Y_i = \phi_0 + \phi_1 \cdot$ $+ u_i$, ie. You incorrectly specified your initial regression.

A) → Calculate $\hat{\beta}_1$ under the assumption that the initial specification is correct

B) → Use the knowledge of Var() and Cov() to write the $\hat{\beta}_1$ formula in ~~their~~ terms of ~~expectation and~~ covariance and variance

C) → Given we know that the true data generating process is $Y_i = \phi_0 + \phi_1 X_{1i} + \phi_2 X_{2i}$, show that our initial $\hat{\beta}_1$ is a biased estimate of ϕ_1

→ Prove that the bias can be characterised:

$$E(\tilde{\phi}_1) = \beta_1 + \beta_2 \frac{Cov(X_{1i}, X_2)}{Var(X_1)}$$

→ What does the direction of the bias depend on?

② Consistent and unbiased estimation:

You have a standard SLR, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$. You can assume $E[u_i|X] = 0$, $Var(u_i|X) = \sigma^2$ and $X = (X_1, \dots, X_n)^T$ } ie. n observations.

A) Consider the estimator $\tilde{\beta}_1$ of β_1 (Not OLS), $\tilde{\beta}_1 = \dfrac{Y_3 + Y_1 - 2Y_2}{X_3 + X_1 - 2X_2}$

→ Is $\tilde{\beta}_1$ consistent?

→ Is $\tilde{\beta}_1$ unbiased?